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Description Estimation and testing methods for dependently truncated data. Semi-parametric methods are based on Emura et al. (2011)<Stat Sinica 21:349-67>, Emura & Wang (2012)<doi:10.1016/j.jmva.2012.03.012>, and Emura & Murotani (2015)<doi:10.1007/s11749-015-0432-8>. Parametric approaches are based on Emura & Konno (2012)<doi:10.1007/s00362-014-0626-2> and Emura & Pan (2017)<doi:10.1007/s00362-017-0947-z>. A regression approach is based on Emura & Wang (2016)<doi:10.1007/s10463-015-0526-9>. Quasi-independence tests are based on Emura & Wang (2010)<doi:10.1016/j.jmva.2009.07.006>. Right-truncated data for Japanese male centenarians are given by Emura & Murotani (2015)<doi:10.1007/s11749-015-0432-8>.

License GPL-2

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`depend.truncation-package`

Statistical Methods for the Analysis of Dependently Truncated Data

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Description

Estimation and testing methods for dependently truncated data. Semi-parametric methods are based on Emura et al. (2011), Emura & Wang (2012), and Emura & Murotani (2015). Parametric approaches are based on Emura & Konno (2012) and Emura & Pan (2017). A regression approach is based on Emura & Wang (2016). Quasi-independence tests are based on Emura & Wang (2010). Right-truncated data for Japanese male centenarians are given by Emura & Murotani (2015).

Details

Package:	depend.truncation
Type:	Package
Version:	3.0
Date:	2018-02-27
License:	GPL-2

Author(s)

Takeshi Emura <takeshiemura@gmail.com>

References

Emura T, Wang W (2010), Testing quasi-independence for truncation data. Journal of Multivariate Analysis 101, 223-239.

Emura T, Wang W, Hung HN (2011), Semiparametric Inference for Copula Models for Truncated Data, Stat Sinica 21: 349-367.

Emura T, Konno Y (2012), Multivariate Normal Distribution Approaches for Dependently Truncated Data. Statistical Papers 53 (No.1), 133-149.

Emura T, Wang W (2012), Nonparametric Maximum Likelihood Estimation for Dependent Truncation Data Based on Copulas, Journal of Multivariate Analysis 110: 171-88

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Emura T, Wang W (2016), Semiparametric Inference for an Accelerated Failure Time Model with Dependent Truncation, Ann Inst Stat Math 68 (5): 1073-94.

Emura T, Pan CH (2017), Parametric likelihood inference and goodness-of-fit for dependently left-truncated data, a copula-based approach, Statistical Papers, doi:10.1007/s00362-017-0947-z.

`centenarian` *Japanese Centenarians Data*

Description

The number of deaths at each year (1963-1980) for Japanese male centenarians (Table 2 of Emura and Murotani (2015)). See also the original reference Sibuya & Hanayama (2004).

Usage

```
data("centenarian")
```

Format

A data frame with 21 observations on the following 19 variables.

`X` :the age at death

`X1963` :the number of deaths between 1963 and 1964

`X1964` :the number of deaths between 1964 and 1965

`X1965` :the number of deaths between 1965 and 1966

`X1966` :the number of deaths between 1966 and 1967

`X1967` :the number of deaths between 1967 and 1968

`X1968` :the number of deaths between 1968 and 1969

`X1969` :the number of deaths between 1969 and 1970

`X1970` :the number of deaths between 1970 and 1971

`X1971` :the number of deaths between 1971 and 1972

`X1972` :the number of deaths between 1972 and 1973

`X1973` :the number of deaths between 1973 and 1974

`X1974` :the number of deaths between 1974 and 1975

`X1975` :the number of deaths between 1975 and 1976

`X1976` :the number of deaths between 1976 and 1977

`X1977` :the number of deaths between 1977 and 1978

`X1978` :the number of deaths between 1978 and 1979

`X1979` :the number of deaths between 1979 and 1980

`X1980` :the number of deaths between 1980 and 1981

References

Sibuya M, Hanayama N (2004), Estimation of Human Longevity Distribution Based on Tabulated Statistics. Proceeding of ISM 52: 117-34

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Examples

```
## Below is the centenarians data analysis of Emura & Murotani (2015) ##
data(centenarian)
Death=centenarian[,1]
Year=1963:1980
data.mat=centenarian[,-1]

X=T=NULL
for(i in 1:length(Death)){
  for(j in 1:length(Year)){
    X=c( X,rep(Death[i],data.mat[i,j]) )
    T=c( T,rep(Year[j]-i+1,data.mat[i,j]) )  ### T= Year at age 100.5 ###
  }
}

x.trunc=X
z.trunc=max(Year)+0.5-T+100

m=length(x.trunc)
d=rep(1,m)

set.seed(1)
x.trunc=x.trunc+runif(length(x.trunc),min=-0.01,max=0)
z.trunc=z.trunc+runif(length(z.trunc),min=0,max=0.01)

### Copula-based estimator ####
## CHAIEB.Frank(x.trunc, z.trunc, d, a = 1/10)
```

CHAIEB.Clayton *Semi-parametric Inference under the Clayton Copula with Dependent Truncation*

Description

A copula-based estimation based on dependent truncation data under the Clayton copula model(Lakhal-Chaieb, Rivest & Abdous 2006; Emura & Murotani 2015). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Clayton copula.

Usage

```
CHAIEB.Clayton(x.trunc, z.trunc, d, a = 1/10, plotX = TRUE, plotY = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=z.trunc
z.trunc	vector of variables satisfying x.trunc<=z.trunc
d	censoring indicator(0=censoring,1=failure) for z.trunc
a	tuning parameter adjusting for small the risk sets (pp.360-361: Emura, Wang & Hung 2011 Sinica)
plotX	if TRUE, plot the distribution function of X
plotY	if TRUE, plot the survival function of Y

Details

The function produces the moment-based estimate for the marginal distributions and the estimate of the association parameter under the Clayton copula model. The method can handle right-censoring for Y in which Z=min(Y, C) and I(Y<=C) are observed with censoring variable C.

Value

alpha	association parameter
tau	Kendall's tau between X and Y
c	inclusion probability, defined by c=Pr(X<=Z)
Fx	marginal distribution function of X at at (ordered) observed points of X
Sy	margianl survival function of Y at (ordered) observed points of Y

Author(s)

Takeshi EMURA

References

Chaieb L, Rivest LP, Abdous B (2006), Estimating Survival Under a Dependent Truncation, Biometrika 93: 655-669.

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Examples

```
###########  AIDS data of Kalbfleisch & Lawless (1989, JASA) #############
inf1=c(23,38,38,36,27,45,23,48,25,42,33,45,33,39,34,48,50,35,26,43,3,53,40,53,
28,34,42,66,19,21,37,33,31,48,32,43,17,64,58,67,49,67,35,12,19,60,53,56,65,53)
inf2=c(36,12,50,45,33,55,37,46,62,57,34,56,57,64,57,42,54,28,36,66,19,45,53,67,
68,54,40,60,54,42,74,71,43,61,68,52,75,46,62,67,48,68,58,55,70,33,56,47,64,15)
inf3=c(41,35,67,35,71,67,36,69,45,76,66,22,74,49,42,15,29,62,65,75,65,69,62,56,
52,82,46,75,27,56,70,49,66,73,76,43,50,41,49,68,39,67,61,82,69,65,56,59,57,57)
inf4=c(46,68,76,64,50,59,80,46,78,26,62,19,22,26,76,27,62,75,76,57,58,59,8,41,
70,57,58,63,37,75,58,39,38,73,72,41,56,50,79,83,76,29,17,69,86,29,65,75,74,42)
inf5=c(65,61,84,41,83,58,45,83,80,84,59,85,59,65,47,64,73,81,79,36,67,87,85,29,
72,77,72,67,53,54,17,61,65,48,57,29,36,30,45,40,75,43,76,66,86,57,75,71,51)
inf6=c(60,41,53,48,36,72,60,36,48,55,58,60,89,37,82,41,68,71,63,49,37,60,78,52,
60,85,68,37,39,22,12,63,80,45,47,85,29,60,84,70,61,69,77,63)
x1=c(27,14,15,18,28,10,34,17,34,17,29,17,29,23,29,15,13,29,38,21,61,12,25,12,38,
32,24,0,48,46,30,34,37,21,37,26,53,6,13,4,22,4,37,60,53,13,20,17,8,20)
x2=c(38,62,24,29,41,19,37,28,13,18,41,19,18,11,18,33,21,48,40,10,58,32,24,11,10,
25,39,19,25,37,5,8,37,20,13,29,6,35,19,14,33,13,23,27,12,49,26,35,18,68)
x3=c(42,48,16,48,12,16,47,14,38,8,18,62,10,35,42,69,55,22,20,10,20,16,23,30,34,4,
40,11,59,30,16,37,20,13,10,43,36,46,38,19,48,20,26,5,18,23,32,29,31,31)
x4=c(42,20,12,24,38,29,8,43,11,63,27,70,67,63,13,62,27,15,14,33,32,32,83,50,21,
34,33,29,55,17,34,53,54,19,20,52,37,43,14,10,17,64,76,25,8,65,29,19,20,52)
x5=c(29,33,10,53,11,36,49,12,15,11,36,10,36,31,49,32,23,15,17,60,29,9,11,67,24,
19,24,29,43,43,80,36,32,49,40,68,61,68,53,58,23,55,22,32,12,41,23,27,47)
x6=c(38,57,46,51,63,27,39,63,51,44,41,39,10,63,18,59,32,29,37,51,63,40,22,48,40,
15,33,64,62,79,89,38,21,56,54,16,72,41,17,31,40,32,24,38)

t=c(inf1,inf2,inf3,inf4,inf5,inf6) #### the month of infection with 1=January 1978 ####
x=c(x1,x2,x3,x4,x5,x6) #### the duration of the incubation period (month) ####
y=102-t #### 102 is the study period (month) ####

######### Breaking ties by adding small noise to the data ##########
set.seed(1)
x=x+runif(293,min=-0.4,max=0.4)
y=y+runif(293,min=-0.4,max=0.4)
x[x<=0]=runif(1,0,0.4)

x.trunc=x
z.trunc=y
d=rep(1,length(x.trunc)) ### all data is not censored ###

CHAIEB.Clayton(x.trunc,z.trunc,d)[c(1,2,3)]
######### The same numerical results as Table 3 of Emura et al. (2011) ###########
```

```
### Below is the small data analysis in Section 3.2 of Emura and Murotani (2015)###
x.trunc=c(1,2,4)
z.trunc=c(3,5,6)
d=c(1,1,1)
CHAIEB.Clayton(x.trunc, z.trunc, d, a = 1/10)
```

CHAIEB.Frank *Semi-parametric Inference under the Frank Copula with Dependent Truncation*

Description

A copula-based estimation based on dependent truncation data under the Frank copula model(Lakhal-Chaieb, Rivest & Abdous 2006; Emura & Murotani 2015). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Frank copula.

Usage

```
CHAIEB.Frank(x.trunc, z.trunc, d, a = 1/10, plotX = TRUE, plotY = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=z.trunc
z.trunc	vector of variables satisfying x.trunc<=z.trunc
d	censoring indicator(0=censoring,1=failure) for z.trunc
a	tuning parameter adjusting for small the risk sets (pp.360-361: Emura, Wang & Hung 2011 Sinica)
plotX	if TRUE, plot the distribution function of X
plotY	if TRUE, plot the survival function of Y

Details

The function produces the moment-based estimate for the marginal distributions and the estimate of the association parameter under the Frank copula model. The method can handle right-censoring for Y in which Z=min(Y, C) and I(Y<=C) are observed with censoring variable C.

Value

alpha	association parameter
tau	Kendall's tau between X and Y
c	inclusion probability, defined by c=Pr(X<=Z)
Fx	marginal distribution function of X at at (ordered) observed points of X
Sy	margianl survival function of Y at (ordered) observed points of Y

Author(s)

Takeshi EMURA

References

Chaieb L, Rivest LP, Abdous B (2006), Estimating Survival Under a Dependent Truncation, Biometrika 93: 655-669.

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Examples

```
###########  AIDS data of Kalbfleisch & Lawless (1989, JASA) #############
inf1=c(23,38,38,36,27,45,23,48,25,42,33,45,33,39,34,48,50,35,26,43,3,53,40,53,
28,34,42,66,19,21,37,33,31,48,32,43,17,64,58,67,49,67,35,12,19,60,53,56,65,53)
inf2=c(36,12,50,45,33,55,37,46,62,57,34,56,57,64,57,42,54,28,36,66,19,45,53,67,
68,54,40,60,54,42,74,71,43,61,68,52,75,46,62,67,48,68,58,55,70,33,56,47,64,15)
inf3=c(41,35,67,35,71,67,36,69,45,76,66,22,74,49,42,15,29,62,65,75,65,69,62,56,
52,82,46,75,27,56,70,49,66,73,76,43,50,41,49,68,39,67,61,82,69,65,56,59,57,57)
inf4=c(46,68,76,64,50,59,80,46,78,26,62,19,22,26,76,27,62,75,76,57,58,59,8,41,
70,57,58,63,37,75,58,39,38,73,72,41,56,50,79,83,76,29,17,69,86,29,65,75,74,42)
inf5=c(65,61,84,41,83,58,45,83,80,84,59,85,59,65,47,64,73,81,79,36,67,87,85,29,
72,77,72,67,53,54,17,61,65,48,57,29,36,30,45,40,75,43,76,66,86,57,75,71,51)
inf6=c(60,41,53,48,36,72,60,36,48,55,58,60,89,37,82,41,68,71,63,49,37,60,78,52,
60,85,68,37,39,22,12,63,80,45,47,85,29,60,84,70,61,69,77,63)
x1=c(27,14,15,18,28,10,34,17,34,17,29,17,29,23,29,15,13,29,38,21,61,12,25,12,38,
32,24,0,48,46,30,34,37,21,37,26,53,6,13,4,22,4,37,60,53,13,20,17,8,20)
x2=c(38,62,24,29,41,19,37,28,13,18,41,19,18,11,18,33,21,48,40,10,58,32,24,11,10,
25,39,19,25,37,5,8,37,20,13,29,6,35,19,14,33,13,23,27,12,49,26,35,18,68)
x3=c(42,48,16,48,12,16,47,14,38,8,18,62,10,35,42,69,55,22,20,10,20,16,23,30,34,4,
40,11,59,30,16,37,20,13,10,43,36,46,38,19,48,20,26,5,18,23,32,29,31,31)
x4=c(42,20,12,24,38,29,8,43,11,63,27,70,67,63,13,62,27,15,14,33,32,32,83,50,21,
34,33,29,55,17,34,53,54,19,20,52,37,43,14,10,17,64,76,25,8,65,29,19,20,52)
x5=c(29,33,10,53,11,36,49,12,15,11,36,10,36,31,49,32,23,15,17,60,29,9,11,67,24,
19,24,29,43,43,80,36,32,49,40,68,61,68,53,58,23,55,22,32,12,41,23,27,47)
x6=c(38,57,46,51,63,27,39,63,51,44,41,39,10,63,18,59,32,29,37,51,63,40,22,48,40,
15,33,64,62,79,89,38,21,56,54,16,72,41,17,31,40,32,24,38)

t=c(inf1,inf2,inf3,inf4,inf5,inf6) #### the month of infection with 1=January 1978 ####
x=c(x1,x2,x3,x4,x5,x6) #### the duration of the incubation period (month) ####
y=102-t #### 102 is the study period (month) ####

######### Breaking ties by adding small noise to the data ##########
set.seed(1)
x=x+runif(293,min=-0.4,max=0.4)
y=y+runif(293,min=-0.4,max=0.4)
x[x<=0]=runif(1,0,0.4)

x.trunc=x
z.trunc=y
d=rep(1,length(x.trunc)) ### all data is not censored ###
```

```
CHAIEB.Frank(x.trunc,z.trunc,d)[c(1,2,3)]
######### The same numerical results as Table 3 of Emura et al. (2011) ###########
```

---

dependAFT.reg *Semiparametric Inference for the AFT regression Model with Dependent Truncation*

Description

Regression estimation for the AFT regression model based on left-truncated and right-censored data (Emura & Wang 2016). The dependency of truncation on lifetime is modeled through the AFT regression form.

Usage

```
dependAFT.reg(t.trunc, y.trunc, d, x1.trunc, initial = c(0, 0), LY = FALSE,
beta1_low = -0.2, beta1_up = 0.2, alpha = 1, epsilon = 1/50)
```

Arguments

t.trunc	vector of left-truncation variables satisfying t.trunc<=y.trunc
y.trunc	vector of lifetime variables satisfying t.trunc<=y.trunc
d	vector of censoring indicators
x1.trunc	vector of 1-dimensional covariates
initial	a pair of initial values for (beta, gamma)
LY	Lai and Ying's estimator for initial values
beta1_low	lower bound for beta
beta1_up	upper bound for beta
alpha	some tuning parameter for optimization, alpha=1 is default
epsilon	some tuning parameter for kernel methods

Details

Only the univariate regression (only one covariate) is allowed.

Value

beta	inference results for beta
gamma	inference results for gamma
beta_LY	the estimator of Lai & Ying (1991)
S2_Minimum	minimum of the objective function
detail	detailed results for minimizing the estimating objective function "optim"

Author(s)

Takeshi Emura

References

Emura T, Wang W (2016), Semiparametric Inference for an Accelerated Failure Time Model with Dependent Truncation, Ann Inst Stat Math 68 (5): 1073-94.

Lai TL, Ying Z (1991), Rank Regression Methods for Left-Truncated and Right-Censored Data. Annals of Statistics 19: 531-556.

Examples

```
y.trunc=c(
  -0.52,  0.22, -1.42,  0.05,  0.32, -1.02, -0.47,  0.10, -0.38, -0.18,  0.97,  0.04, -0.10,
   0.50,  0.57, -0.80, -0.24,  0.07, -0.04,  0.88, -0.52, -0.28, -0.55,  0.53,  0.99, -0.52,
  -0.59, -0.48, -0.07,  0.20, -0.34,  1.00, -0.52)
t.trunc=c(
  -2.05, -0.25, -2.43, -0.32, -0.27, -1.06, -0.95, -0.82, -0.66, -0.28, -1.14, -0.32, -1.19,
  -2.18, -0.45, -1.71, -0.84, -1.93, -1.04, -2.58, -1.97, -2.15, -0.59, -0.74, -1.26, -2.57,
  -2.40, -2.22, -1.52, -0.21, -1.50, -1.99, -1.79)

d=c(1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1,
    0, 1, 1,1)

x1.trunc=c(
  0.27, 0.66, 0.77, 0.21, 0.48, 0.11, 0.69, 0.32, 0.33, 0.43, 0.12, 0.60, 0.13, 0.43, 0.99,
  0.21, 0.93, 0.60, 0.45, 0.41, 0.86, 0.90, 0.76, 0.93, 0.27, 0.13, 0.82, 0.17, 0.63, 0.31,
  0.13, 0.48, 0.32)

### Data analysis in Emura & Wang (2016) ###
# dependAFT.reg(t.trunc,y.trunc,d,x1.trunc,alpha=2,LY=TRUE,beta1_low=-5,beta1_up=5)
dependAFT.reg(t.trunc,y.trunc,d,x1.trunc,LY=FALSE,beta1_low=-5,beta1_up=5)

#### Channing hourse data analysis; Section 5 of Emura & Wang (2016) #####
# library(KMsurv)
# data(channing)
# y.trunc=log(channing$age)
# t.trunc=log(channing$ageentry)
# d=channing$death
# x1.trunc=as.numeric(channing$gender==1)

# dependAFT.reg(t.trunc,y.trunc,d,x1.trunc,beta1_low=-0.2,beta1_up=0.2)
# dependAFT.reg(t.trunc,y.trunc,d,x1.trunc,LY=TRUE,alpha=2,beta1_low=-0.2,beta1_up=0.2)
```

EMURA.Clayton *Semi-parametric Inference under the Clayton Copula with Dependent Truncation*

Description

A copula-based estimation based on dependent truncation data under the Clayton copula model (Emura, Wang & Hung 2011; Emura & Murotani 2015). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Clayton copula.

Usage

```
EMURA.Clayton(x.trunc, z.trunc, d, a = 1/10, plotX = TRUE, plotY = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=z.trunc
z.trunc	vector of variables satisfying x.trunc<=z.trunc
d	censoring indicator(0=censoring,1=failure) for z.trunc
a	tuning parameter adjusting for small the risk sets (pp.360-361: Emura, Wang & Hung 2011 Sinica)
plotX	if TRUE, plot the distribution function of X
plotY	if TRUE, plot the survival function of Y

Details

The function produces the moment-based estimate for the marginal distributions and the estimate of the association parameter under the Clayton copula model. The method can handle right-censoring for Y in which Z=min(Y, C) and I(Y<=C) are observed with censoring variable C.

Value

alpha	association parameter
tau	Kendall's tau between X and Y
alpha	association parameter
c	inclusion probability, defined by c=Pr(X<=Z)
Fx	marginal distribution function of X at at (ordered) observed points of X
Sy	margianl survival function of Y at (ordered) observed points of Y

Author(s)

Takeshi Emura

References

Emura T, Wang W, Hung HN (2011), Semiparametric Inference for Copula Models for Truncated Data, Stat Sinica 21: 349-367.

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Examples

```
############  AIDS data of Kalbfleisch & Lawless (1989, JASA) #############
inf1=c(23,38,38,36,27,45,23,48,25,42,33,45,33,39,34,48,50,35,26,43,3,53,40,53,
28,34,42,66,19,21,37,33,31,48,32,43,17,64,58,67,49,67,35,12,19,60,53,56,65,53)
inf2=c(36,12,50,45,33,55,37,46,62,57,34,56,57,64,57,42,54,28,36,66,19,45,53,67,
68,54,40,60,54,42,74,71,43,61,68,52,75,46,62,67,48,68,58,55,70,33,56,47,64,15)
inf3=c(41,35,67,35,71,67,36,69,45,76,66,22,74,49,42,15,29,62,65,75,65,69,62,56,
52,82,46,75,27,56,70,49,66,73,76,43,50,41,49,68,39,67,61,82,69,65,56,59,57,57)
inf4=c(46,68,76,64,50,59,80,46,78,26,62,19,22,26,76,27,62,75,76,57,58,59,8,41,
70,57,58,63,37,75,58,39,38,73,72,41,56,50,79,83,76,29,17,69,86,29,65,75,74,42)
inf5=c(65,61,84,41,83,58,45,83,80,84,59,85,59,65,47,64,73,81,79,36,67,87,85,29,
72,77,72,67,53,54,17,61,65,48,57,29,36,30,45,40,75,43,76,66,86,57,75,71,51)
inf6=c(60,41,53,48,36,72,60,36,48,55,58,60,89,37,82,41,68,71,63,49,37,60,78,52,
60,85,68,37,39,22,12,63,80,45,47,85,29,60,84,70,61,69,77,63)
x1=c(27,14,15,18,28,10,34,17,34,17,29,17,29,23,29,15,13,29,38,21,61,12,25,12,38,
32,24,0,48,46,30,34,37,21,37,26,53,6,13,4,22,4,37,60,53,13,20,17,8,20)
x2=c(38,62,24,29,41,19,37,28,13,18,41,19,18,11,18,33,21,48,40,10,58,32,24,11,10,
25,39,19,25,37,5,8,37,20,13,29,6,35,19,14,33,13,23,27,12,49,26,35,18,68)
x3=c(42,48,16,48,12,16,47,14,38,8,18,62,10,35,42,69,55,22,20,10,20,16,23,30,34,4,
40,11,59,30,16,37,20,13,10,43,36,46,38,19,48,20,26,5,18,23,32,29,31,31)
x4=c(42,20,12,24,38,29,8,43,11,63,27,70,67,63,13,62,27,15,14,33,32,32,83,50,21,
34,33,29,55,17,34,53,54,19,20,52,37,43,14,10,17,64,76,25,8,65,29,19,20,52)
x5=c(29,33,10,53,11,36,49,12,15,11,36,10,36,31,49,32,23,15,17,60,29,9,11,67,24,
19,24,29,43,43,80,36,32,49,40,68,61,68,53,58,23,55,22,32,12,41,23,27,47)
x6=c(38,57,46,51,63,27,39,63,51,44,41,39,10,63,18,59,32,29,37,51,63,40,22,48,40,
15,33,64,62,79,89,38,21,56,54,16,72,41,17,31,40,32,24,38)

t=c(inf1,inf2,inf3,inf4,inf5,inf6) #### the month of infection with 1=January 1978 ####
x=c(x1,x2,x3,x4,x5,x6) #### the duration of the incubation period (month) ####
y=102-t #### 102 is the study period (month) ####

######### Breaking ties by adding small noise to the data ##########
set.seed(1)
x=x+runif(293,min=-0.4,max=0.4)
y=y+runif(293,min=-0.4,max=0.4)
x[x<=0]=runif(1,0,0.4)

x.trunc=x
z.trunc=y
d=rep(1,length(x.trunc)) ### all data is not censored ###

EMURA.Clayton(x.trunc,z.trunc,d)[c(1,2,3)]
######### The same numerical results as Table 3 of Emura et al. (2011) ###########
```

EMURA.Frank *Semi-parametric Inference under the Frank Copula with Dependent Truncation*

Description

A copula-based estimation based on dependent truncation data under the Frank copula model (Emura, Wang & Hung 2011; Emura & Murotani 2015). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Frank copula.

Usage

```
EMURA.Frank(x.trunc, z.trunc, d, a = 1/10, plotX = TRUE, plotY = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=z.trunc
z.trunc	vector of variables satisfying x.trunc<=z.trunc
d	censoring indicator(0=censoring,1=failure) for z.trunc
a	tuning parameter adjusting for small the risk sets (pp.360-361: Emura, Wang & Hung 2011 Sinica)
plotX	if TRUE, plot the distribution function of X
plotY	if TRUE, plot the survival function of Y

Details

The function produces the moment-based estimate for the marginal distributions and the estimate of the association parameter under the Frank copula model. The method can handle right-censoring for Y in which Z=min(Y, C) and I(Y<=C) are observed with censoring variable C.

Value

alpha	association parameter
tau	Kendall's tau between X and Y
c	inclusion probability, defined by c=Pr(X<=Z)
Fx	marginal distribution function of X at at (ordered) observed points of X
Sy	margianl survival function of Y at (ordered) observed points of Y

Author(s)

Takeshi EMURA

References

Emura T, Wang W, Hung HN (2011), Semiparametric Inference for Copula Models for Truncated Data, Stat Sinica 21: 349-367.

Emura T, Murotani K (2015), An Algorithm for Estimating Survival Under a Copula-based Dependent Truncation Model, TEST 24 (No.4): 734-751.

Examples

```
###########  AIDS data of Kalbfleisch & Lawless (1989, JASA) #############
inf1=c(23,38,38,36,27,45,23,48,25,42,33,45,33,39,34,48,50,35,26,43,3,53,40,53,
28,34,42,66,19,21,37,33,31,48,32,43,17,64,58,67,49,67,35,12,19,60,53,56,65,53)
inf2=c(36,12,50,45,33,55,37,46,62,57,34,56,57,64,57,42,54,28,36,66,19,45,53,67,
68,54,40,60,54,42,74,71,43,61,68,52,75,46,62,67,48,68,58,55,70,33,56,47,64,15)
inf3=c(41,35,67,35,71,67,36,69,45,76,66,22,74,49,42,15,29,62,65,75,65,69,62,56,
52,82,46,75,27,56,70,49,66,73,76,43,50,41,49,68,39,67,61,82,69,65,56,59,57,57)
inf4=c(46,68,76,64,50,59,80,46,78,26,62,19,22,26,76,27,62,75,76,57,58,59,8,41,
70,57,58,63,37,75,58,39,38,73,72,41,56,50,79,83,76,29,17,69,86,29,65,75,74,42)
inf5=c(65,61,84,41,83,58,45,83,80,84,59,85,59,65,47,64,73,81,79,36,67,87,85,29,
72,77,72,67,53,54,17,61,65,48,57,29,36,30,45,40,75,43,76,66,86,57,75,71,51)
inf6=c(60,41,53,48,36,72,60,36,48,55,58,60,89,37,82,41,68,71,63,49,37,60,78,52,
60,85,68,37,39,22,12,63,80,45,47,85,29,60,84,70,61,69,77,63)
x1=c(27,14,15,18,28,10,34,17,34,17,29,17,29,23,29,15,13,29,38,21,61,12,25,12,38,
32,24,0,48,46,30,34,37,21,37,26,53,6,13,4,22,4,37,60,53,13,20,17,8,20)
x2=c(38,62,24,29,41,19,37,28,13,18,41,19,18,11,18,33,21,48,40,10,58,32,24,11,10,
25,39,19,25,37,5,8,37,20,13,29,6,35,19,14,33,13,23,27,12,49,26,35,18,68)
x3=c(42,48,16,48,12,16,47,14,38,8,18,62,10,35,42,69,55,22,20,10,20,16,23,30,34,4,
40,11,59,30,16,37,20,13,10,43,36,46,38,19,48,20,26,5,18,23,32,29,31,31)
x4=c(42,20,12,24,38,29,8,43,11,63,27,70,67,63,13,62,27,15,14,33,32,32,83,50,21,
34,33,29,55,17,34,53,54,19,20,52,37,43,14,10,17,64,76,25,8,65,29,19,20,52)
x5=c(29,33,10,53,11,36,49,12,15,11,36,10,36,31,49,32,23,15,17,60,29,9,11,67,24,
19,24,29,43,43,80,36,32,49,40,68,61,68,53,58,23,55,22,32,12,41,23,27,47)
x6=c(38,57,46,51,63,27,39,63,51,44,41,39,10,63,18,59,32,29,37,51,63,40,22,48,40,
15,33,64,62,79,89,38,21,56,54,16,72,41,17,31,40,32,24,38)

t=c(inf1,inf2,inf3,inf4,inf5,inf6) #### the month of infection with 1=January 1978 ####
x=c(x1,x2,x3,x4,x5,x6) #### the duration of the incubation period (month) ####
y=102-t #### 102 is the study period (month) ####

######### Breaking ties by adding small noise to the data ##########
set.seed(1)
x=x+runif(293,min=-0.4,max=0.4)
y=y+runif(293,min=-0.4,max=0.4)
x[x<=0]=runif(1,0,0.4)

x.trunc=x
z.trunc=y
d=rep(1,length(x.trunc)) ### all data is not censored ###

EMURA.Frank(x.trunc,z.trunc,d)[c(1,2,3)]
######### The same numerical results as Table 3 of Emura et al. (2011) ###########
```

`Logrank.stat` *The weighted log-rank statistics for testing quasi-independence (without ties in data)*

Description

The three log-rank statistics (L_0, L_1, and L_log) corresponding to 3 different weights.

Usage

```
Logrank.stat(x.trunc, z.trunc, d)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=z.trunc
z.trunc	vector of variables satisfying x.trunc<=z.trunc
d	censoring indicator(0=censoring,1=failure) for z.trunc

Details

If there is no tie in the data, the function "Logrank.stat.tie" and "Logrank.stat" give identical results. However, "Logrank.stat" is computationally more efficient. The simulations of Emura & Wang (2010) are based on "Logrank.stat" since simulated data are generated from continuous distributions. The real data analyses of Emura & Wang (2010) are based on "Logrank.stat.tie" since there are many ties in the data.

Value

L0	Logrank statistics (most powerfull to detect the Clayton copula type dependence)
L1	Logrank statistics (most powerfull to detect the Frank copula type dependence)
Llog	Logrank statistics (most powerfull to detect the Gumbel copula type dependence)

Author(s)

Takeshi Emura

References

Emura T, Wang W (2010) Testing quasi-independence for truncation data. Journal of Multivariate Analysis 101, 223-239

Examples

```
x.trunc=c(10,5,7,1,3,9)
z.trunc=c(12,11,8,6,4,13)
d=c(1,1,1,1,0,1)
Logrank.stat(x.trunc,z.trunc,d)
```

`Logrank.stat.tie` *The weighted log-rank statistics for testing quasi-independence (with ties in data)*

Description

The three log-rank statistics (L_0, L_1, and L_log) corresponding to 3 different weights.

Usage

```
Logrank.stat.tie(x.trunc, z.trunc, d)
```

Arguments

`x.trunc`	vector of variables satisfying x.trunc<=z.trunc
`z.trunc`	vector of variables satisfying x.trunc<=z.trunc
`d`	censoring indicator(0=censoring,1=failure) for z.trunc

Details

If there is no tie in the data, the function "Logrank.stat.tie" and "Logrank.stat" give identical results. However, "Logrank.stat" is computationally more efficient. The simulations of Emura & Wang (2010) are based on "Logrank.stat" since simulated data are generated from continuous distributions. The real data analyses of Emura & Wang (2010) are based on "Logrank.stat.tie" since there are many ties in the data.

Value

`L0`	Logrank statistics (most powerfull to detect the Clayton copula type dependence)
`L1`	Logrank statistics (most powerfull to detect the Frank copula type dependence)
`Llog`	Logrank statistics (most powerfull to detect the Gumbel copula type dependence)

Author(s)

Takeshi Emura

References

Emura T, Wang W (2010) Testing quasi-independence for truncation data. Journal of Multivariate Analysis 101, 223-239

Examples

```
x.trunc=c(10,5,7,1,3,9)
z.trunc=c(12,11,8,6,4,13)
d=c(1,1,1,1,0,1)
Logrank.stat.tie(x.trunc,z.trunc,d)
Logrank.stat(x.trunc,z.trunc,d) ## since there is no tie, the results are the same.
```

NPMLE.Frank *Semiparametric Inference under the Frank Copula with Dependent Truncation*

Description

Nonparametric maximum likelihood estimation for dependent truncation data under the Frank copula models (Emura & Wang, 2012). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Frank copula.

Usage

```
NPMLE.Frank(x.trunc, y.trunc,
x.fix = median(x.trunc), y.fix = median(y.trunc), plotX = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=y.trunc
y.trunc	vector of variables satisfying x.trunc<=y.trunc
x.fix	vector of fixed points at which marginal distribution function of X is calculated
y.fix	vector of fixed points at which marginal survival function of Y are calculated
plotX	if TRUE, the culumative distribution function for X is plotted

Details

The function produces the nonparametric maximum likelihood estimate (NPMLE) for the marginal distributions and the estimate of the association parameter under the Frank copula model. The method follows Emura & Wang (2012). The maximization of the likelihood (L) is conducted by minimizing -logL by "nlm". Standard errors are calculated from the diagonal elements in the observed Fisher information matrix, which is obtained from the output of the "nlm". The method can handle ties but cannot handle right-censoring.

Value

alpha	estimate of association parameter
alpha_se	standard error of the estimate of association parameter
Hx	estimate of the marginal cumulative reverse-hazard function for X
Hx_se	standard error of the estimate of the marginal cumulative reverse-hazard function for X

Ay	estimate of the marginal cumulative hazard function for Y
Ay_se	standard error of the estimate of the marginal cumulative hazard function for Y
Fx	estimate of the marginal distribution function for X
Fx_se	standard error of the estimate of the marginal distribution function for X
Sy	estimate of the marginal survival function for Y
Sy_se	standard error of the estimate of the marginal survival function for Y
conv	an integer indicating why the optimization process terminated in "nlm": If conv=1, the maximization of the likelihood function is properly done. Please refer R function "nlm" for more details.
iteration	the number of iterations until convergence
Grad	L_2 norm for the gradient vector at the solution (close to zero if the solution is proper)
MinEigen	Minimum eigenvalue of the Hessian matrix at the solution (positive if the solution is proper)

Author(s)

Takeshi EMURA

References

Emura T, Wang W (2012) Nonparametric maximum likelihood estimation for dependent truncation data based on copulas, Journal of Multivariate Analysis 110, 171-88

Examples

```
x.trunc=c(0.53, 0.43, 1.63, 0.98, 0.62)
y.trunc=c(1.20, 4.93, 2.09, 2.57, 1.52)
NPMLE.Frank(x.trunc, y.trunc, x.fix =1, y.fix = 2, plotX = TRUE)
```

NPMLE.Indep *Semiparametric Inference under the Independence Copula with Dependent Truncation*

Description

Nonparametric maximum likelihood estimation for dependent truncation data under the independence copula models (Emura & Wang, 2012). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the independence copula.

Usage

```
NPMLE.Indep(x.trunc, y.trunc,
x.fix = median(x.trunc), y.fix = median(y.trunc), plotX = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=y.trunc
y.trunc	vector of variables satisfying x.trunc<=y.trunc
x.fix	vector of fixed points at which marginal distribution function of X is calculated
y.fix	vector of fixed points at which marginal survival function of Y are calculated
plotX	if TRUE, the culumative distribution function for X is plotted

Details

The function produces the nonparametric maximum likelihood estimate (NPMLE) for the marginal distributions and the estimate of the association parameter under the independence copula model. The method follows Emura & Wang (2012). The maximization of the likelihood (L) is conducted by minimizing -logL by "nlm". Standard errors are calculated from the diagonal elements in the observed Fisher information matrix, which is obtained from the output of the "nlm". The method can handle ties but cannot handle right-censoring.

Value

Hx	estimate of the marginal cumulative reverse-hazard function for X
Hx_se	standard error of the estimate of the marginal cumulative reverse-hazard function for X
Ay	estimate of the marginal cumulative hazard function for Y
Ay_se	standard error of the estimate of the marginal cumulative hazard function for Y
Fx	estimate of the marginal distribution function for X
Fx_se	standard error of the estimate of the marginal distribution function for X
Sy	estimate of the marginal survival function for Y
Sy_se	standard error of the estimate of the marginal survival function for Y
conv	an integer indicating why the optimization process terminated in "nlm": If conv=1, the maximization of the likelihood function is properly done. Please refer R function "nlm" for more details.
Grad	L_2 norm for the gradient vector at the solution (close to zero if the solution is proper)
MinEigen	Minimum eigenvalue of the Hessian matrix at the solution (positive if the solution is proper)

Author(s)

Takeshi EMURA

References

Emura T, Wang W (2012) Nonparametric maximum likelihood estimation for dependent truncation data based on copulas, Journal of Multivariate Analysis 110, 171-88

Examples

```
##### Generate independent truncated data with unit exponential marginals #####
set.seed(1)
m=35 #### sample size ####
x.trunc=y.trunc=numeric(m)
l=1
while(l<=m){
  x=rexp(1,rate=1)
  y=rexp(1,rate=1)
  if((x<y)){
  x.trunc[l]=x;y.trunc[l]=y
  l=l+1
  }
}

NPMLE.Indep(x.trunc, y.trunc, x.fix =1, y.fix = 1, plotX = TRUE)

#### compare the above estimate with the true value #####
c(Fx_true=1-exp(-1),Sy_true=exp(-1))
```

`NPMLE.Normal` *Semiparametric Inference under the Normal Copula with Dependent Truncation*

Description

Nonparametric maximum likelihood estimation for dependent truncation data under the normal(Gaussian) copula models (Emura & Wang, 2012). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the normal(Gaussian) copula.

Usage

```
NPMLE.Normal(x.trunc, y.trunc,
x.fix = median(x.trunc), y.fix = median(y.trunc), plotX = TRUE)
```

Arguments

`x.trunc`	vector of variables satisfying x.trunc<=y.trunc
`y.trunc`	vector of variables satisfying x.trunc<=y.trunc
`x.fix`	vector of fixed points at which marginal distribution function of X is calculated
`y.fix`	vector of fixed points at which marginal survival function of Y are calculated
`plotX`	if TRUE, the culumative distribution function for X is plotted

Details

The function produces the nonparametric maximum likelihood estimate (NPMLE) for the marginal distributions and the estimate of the association parameter under the normal(Gaussian) copula model. The method follows Emura & Wang (2012). The maximization of the likelihood (L) is conducted by minimizing -logL by "nlm". Standard errors are calculated from the diagonal elements in the observed Fisher information matrix, which is obtained from the output of the "nlm". The method can handle ties but cannot handle right-censoring.

Value

`alpha`	estimate of association parameter
`alpha_se`	standard error of the estimate of association parameter
`Hx`	estimate of the marginal cumulative reverse-hazard function for X
`Hx_se`	standard error of the estimate of the marginal cumulative reverse-hazard function for X
`Ay`	estimate of the marginal cumulative hazard function for Y
`Ay_se`	standard error of the estimate of the marginal cumulative hazard function for Y
`Fx`	estimate of the marginal distribution function for X
`Fx_se`	standard error of the estimate of the marginal distribution function for X
`Sy`	estimate of the marginal survival function for Y
`Sy_se`	standard error of the estimate of the marginal survival function for Y
`conv`	an integer indicating why the optimization process terminated in "nlm": If conv=1, the maximization of the likelihood function is properly done. Please refer R function "nlm" for more details.
`iteration`	the number of iterations until convergence
`Grad`	L_2 norm for the gradient vector at the solution (close to zero if the solution is proper)
`MinEigen`	Minimum eigenvalue of the Hessian matrix at the solution (positive if the solution is proper)

Author(s)

Takeshi EMURA

References

Emura T, Wang W (2012) Nonparametric maximum likelihood estimation for dependent truncation data based on copulas, Journal of Multivariate Analysis 110, 171-88

Examples

```
x.trunc=c(0.53, 0.43, 1.63, 0.98, 0.62)
y.trunc=c(1.20, 4.93, 2.09, 2.57, 1.52)
# NPMLE.Normal(x.trunc, y.trunc, x.fix =1, y.fix = 2, plotX = TRUE)
```

NPMLE.Plackett *Semiparametric Inference under the Plackett Copula with Dependent Truncation*

Description

Nonparametric maximum likelihood estimation for dependent truncation data under the Plackett copula models (Emura & Wang, 2012). The forms of the marginal distributions for X and Y are completely unspecified, but the parametric form of copula is specified as the Plackett copula.

Usage

```
NPMLE.Plackett(x.trunc, y.trunc,
x.fix = median(x.trunc), y.fix = median(y.trunc), plotX = TRUE)
```

Arguments

x.trunc	vector of variables satisfying x.trunc<=y.trunc
y.trunc	vector of variables satisfying x.trunc<=y.trunc
x.fix	vector of fixed points at which marginal distribution function of X is calculated
y.fix	vector of fixed points at which marginal survival function of Y are calculated
plotX	if TRUE, the culumative distribution function for X is plotted

Details

The function produces the nonparametric maximum likelihood estimate (NPMLE) for the marginal distributions and the estimate of the association parameter under the Plackett copula model. The method follows Emura & Wang (2012). The maximization of the likelihood (L) is conducted by minimizing -logL by "nlm". Standard errors are calculated from the diagonal elements in the observed Fisher information matrix, which is obtained from the output of the "nlm". The method can handle ties but cannot handle right-censoring.

Value

alpha	estimate of association parameter
alpha_se	standard error of the estimate of association parameter
Hx	estimate of the marginal cumulative reverse-hazard function for X
Hx_se	standard error of the estimate of the marginal cumulative reverse-hazard function for X
Ay	estimate of the marginal cumulative hazard function for Y
Ay_se	standard error of the estimate of the marginal cumulative hazard function for Y
Fx	estimate of the marginal distribution function for X
Fx_se	standard error of the estimate of the marginal distribution function for X
Sy	estimate of the marginal survival function for Y

Sy_se	standard error of the estimate of the marginal survival function for Y
conv	an integer indicating why the optimization process terminated in "nlm": If conv=1, the maximization of the likelihood function is properly done. Please refer R function "nlm" for more details.
iteration	the number of iterations until convergence
Grad	L_2 norm for the gradient vector at the solution (close to zero if the solution is proper)
MinEigen	Minimum eigenvalue of the Hessian matrix at the solution (positive if the solution is proper)

Author(s)

Takeshi EMURA

References

Emura T, Wang W (2012) Nonparametric maximum likelihood estimation for dependent truncation data based on copulas, Journal of Multivariate Analysis 110, 171-88

Examples

```
##### Generate truncated data from Plackett copula with unit exponential marginals #####
set.seed(1)
m=25 #### sample size ####
alpha_true=1/5.11 #### Kendall's tau on (X,Y) = 0.5 ####
x.trunc=y.trunc=numeric(m)
l=1
while(l<=m){
  u=runif(1,min=0,max=1)
  v=runif(1,min=0,max=1)
  K=function(w){
    A=sqrt( (1+(alpha_true-1)*(u+w))^2-4*alpha_true*(alpha_true-1)*u*w )
    2*v+(  1+(alpha_true-1)*u-(alpha_true+1)*w  )/A-1
  }
  low=0.00001*u
  up=1-0.00001*v
  for(j in 1:100){ #### Invert K(.) by the bisection method to ####
    mid=(low+up)/2;M<-K(mid);U<-K(up)
    if(M*U>0){up<-mid}else{low<-mid}
  }
  w=mid
  x=-log(1-u);y=-log(w)
  if((x<y)){
  x.trunc[l]=x;y.trunc[l]=y
  l=l+1
  }
}

NPMLE.Plackett(x.trunc, y.trunc, x.fix =1, y.fix = 1, plotX = TRUE)
```

```
#### compare the above estimate with the true value #####
c(alpha_true=alpha_true,Fx_true=1-exp(-1),Sy_true=exp(-1))
```

`PMLE.Clayton.Exponential` *Parametric Inference for Bivariate Exponential Models with Dependent Truncation*

Description

Maximum likelihood estimation (MLE) for dependent truncation data under the Clayton copula with Exponential margins for a bivariate lifetimes (L, X). The truncated data (L_j, X_j), subject to L_j<=X_j for all j=1, ..., n, are used to obtain the MLE for the population parameters of (L, X).

Usage

```
PMLE.Clayton.Exponential(l.trunc, x.trunc, GOF = TRUE,
 Err=3, alpha_max=20,alpha_min=10^-4)
```

Arguments

`l.trunc`	vector of truncation variables satisfying l.trunc<=x.trunc
`x.trunc`	vector of variables satisfying l.trunc<=x.trunc
`GOF`	if TRUE, a goodness-of-fit test statistics is computed
`Err`	tuning parameter in the NR algorithm
`alpha_max`	upper bound for the copula parameter
`alpha_min`	lower bound for the copula parameter

Details

Original paper is submitted for review

Value

`n`	sample size
`alpha`	dependence parameter
`lambda_L`	scale parameter of L
`lambda_X`	scale parameter of X
`mean_X`	Mean lifetime of X, defined as E[X]
`logL`	Maximized log-likelihood
`c`	inclusion probability, defined by c=Pr(L<=X)
`C`	Cramer-von Mises goodness-of-fit test statistics
`K`	Kolmogorov-Smirnov goodness-of-fit test statistics

Author(s)

Takeshi Emura, Chi-Hung Pan

References

Emura T, Pan CH (2017), Parametric likelihood inference and goodness-of-fit for dependently left-truncated data, a copula-based approach, Statistical Papers, doi:10.1007/s00362-017-0947-z.

Examples

```
l.trunc=c(22.207,23.002,23.982,28.551,21.789,17.042,25.997,23.220,18.854,21.857,
    27.321,13.767,23.982,20.110,15.779,26.821,27.934,15.292,28.843,15.985,
    23.580,53.770,21.731,28.844,17.046,16.506,15.696,27.959,13.272,16.482,
    24.210,17.626,27.770,
    18.264,17.694,20.014,13.152,16.886,14.894,15.531,6.951,15.841,14.974,
    38.292,11.204,38.156,26.652,17.101,28.953,18.325,18.391,18.220,15.896,
    16.447,23.642,19.170,23.257,20.428,20.947,28.462,23.210,17.900,46.134,
    39.300,11.768,17.717,
    30.863,22.350,44.976,18.169,30.164,21.822,18.201,22.895,27.189,10.915,
    25.503,12.350,39.869,17.698,26.296,14.091,21.011,11.201,10.757,25.692,
    32.372,13.592,19.102,16.112,53.281,57.298,36.450,19.651,20.755,30.788,20.0,39.62)

x.trunc = c(38.701,49.173,42.409,73.823,46.738,44.071,61.904,39.327,49.828,46.314,
    56.150,50.549,54.930,54.039,49.170,44.795,72.238,107.783,81.609,45.228,
    124.637,64.018,82.957,143.550,43.382,69.644,74.750,32.881,51.483,31.767,
    77.633,63.745,82.965,
    24.818,68.762,68.762,89.100,64.979,65.127,59.289,53.926,79.370,47.385,
    61.395,72.826,53.980,37.220,44.224,50.826,65.460,86.726,43.819,100.605,
    67.615,89.542,60.266,103.580,82.570,87.960,42.385,68.914,95.666,78.135,
    83.643,18.617,92.629,
    42.415,34.346,106.569,20.758,52.003,77.179, 68.934,78.661,165.543,79.547,
    55.009,46.774,124.526,92.504,109.986,101.161,59.422,27.772,33.598,69.038,
    75.222,58.373,105.610,56.158,55.913,83.770,123.468,68.994,101.869,87.627,
    38.790,74.734)

u.min=10
l.trunc=l.trunc[-41]-u.min
x.trunc=x.trunc[-41]-u.min

PMLE.Clayton.Exponential(l.trunc,x.trunc)
```

`PMLE.Clayton.Weibull` *Parametric Inference for Bivariate Weibull Models with Dependent Truncation*

Description

Maximum likelihood estimation (MLE) for dependent truncation data under the Clayton copula with Weibull margins for a bivariate lifetimes (L, X). The truncated data (L_j, X_j), subject to L_j<=X_j for all j=1, ..., n, are used to obtain the MLE for the population parameters of (L, X).

Usage

```
PMLE.Clayton.Weibull(l.trunc, x.trunc, GOF = TRUE,
 Err=2,alpha_max=20,alpha_min=10^-6)
```

Arguments

l.trunc	vector of truncation variables satisfying l.trunc<=x.trunc
x.trunc	vector of variables satisfying l.trunc<=x.trunc
GOF	if TRUE, a goodness-of-fit test statistics is computed
Err	tuning parameter in the NR algorithm
alpha_max	upper bound for the copula parameter
alpha_min	lower bound for the copula parameter

Details

Relevant paper is submitted for review

Value

n	sample size
alpha	dependence parameter
lambda_L	scale parameter of L
lambda_X	scale parameter of X
nu_L	shape parameter of L
nu_X	shape parameter of X
mean_X	Mean lifetime of X, defined as E[X]
logL	Maximized log-likelihood
c	inclusion probability, defined by c=Pr(L<=X)
C	Cramer-von Mises goodness-of-fit test statistics
K	Kolmogorov-Smirnov goodness-of-fit test statistics

Author(s)

Takeshi Emura

References

Emura T, Pan CH (2017), Parametric likelihood inference and goodness-of-fit for dependently left-truncated data, a copula-based approach, Statistical Papers, doi:10.1007/s00362-017-0947-z.

Examples

```
l.trunc=c(22.207,23.002,23.982,28.551,21.789,17.042,25.997,23.220,18.854,21.857,
    27.321,13.767,23.982,20.110,15.779,26.821,27.934,15.292,28.843,15.985,
    23.580,53.770,21.731,28.844,17.046,16.506,15.696,27.959,13.272,16.482,
    24.210,17.626,27.770,
    18.264,17.694,20.014,13.152,16.886,14.894,15.531,6.951,15.841,14.974,
    38.292,11.204,38.156,26.652,17.101,28.953,18.325,18.391,18.220,15.896,
    16.447,23.642,19.170,23.257,20.428,20.947,28.462,23.210,17.900,46.134,
    39.300,11.768,17.717,
    30.863,22.350,44.976,18.169,30.164,21.822,18.201,22.895,27.189,10.915,
    25.503,12.350,39.869,17.698,26.296,14.091,21.011,11.201,10.757,25.692,
    32.372,13.592,19.102,16.112,53.281,57.298,36.450,19.651,20.755,30.788,20.0,39.62)

x.trunc = c(38.701,49.173,42.409,73.823,46.738,44.071,61.904,39.327,49.828,46.314,
    56.150,50.549,54.930,54.039,49.170,44.795,72.238,107.783,81.609,45.228,
    124.637,64.018,82.957,143.550,43.382,69.644,74.750,32.881,51.483,31.767,
    77.633,63.745,82.965,
    24.818,68.762,68.762,89.100,64.979,65.127,59.289,53.926,79.370,47.385,
    61.395,72.826,53.980,37.220,44.224,50.826,65.460,86.726,43.819,100.605,
    67.615,89.542,60.266,103.580,82.570,87.960,42.385,68.914,95.666,78.135,
    83.643,18.617,92.629,
    42.415,34.346,106.569,20.758,52.003,77.179, 68.934,78.661,165.543,79.547,
    55.009,46.774,124.526,92.504,109.986,101.161,59.422,27.772,33.598,69.038,
    75.222,58.373,105.610,56.158,55.913,83.770,123.468,68.994,101.869,87.627,
    38.790,74.734)

u.min=10
l.trunc=l.trunc[-41]-u.min
x.trunc=x.trunc[-41]-u.min

PMLE.Clayton.Weibull(l.trunc,x.trunc)
```

`PMLE.Normal` *Parametric Inference for Bivariate Normal Models with Dependent Truncation*

Description

Maximum likelihood estimation (MLE) for dependent truncation data under the bivariate normal distribution. A bivariate normal distribution is assumed for bivariate random variables (L, X). The truncated data (L_j, X_j), subject to L_j<=X_j for all j=1, ..., n, are used to obtain the MLE for the population parameters of (L, X).

Usage

```
PMLE.Normal(l.trunc, x.trunc, testimator = FALSE,GOF=TRUE)
```

Arguments

l.trunc	vector of truncation variables satisfying l.trunc<=x.trunc
x.trunc	vector of variables satisfying l.trunc<=x.trunc
testimator	if TRUE, testimator is computed instead of MLE
GOF	if TRUE, goodness-of-fit test is performed

Details

PMLE.Normal performs the maximum likelihood estimation for dependently left-truncated data under the bivariate normal distribution. "PMLE.Normal" implements the methodologies developed in Emura T. & Konno Y. (2012, Statistical Papers 53, 133-149)and can produce the maximum likelihood estimates and their standard errors. Furthermore, "PMLE.Normal" tests the independence assumption between truncation variable and variable of interest via likelihood ratio test. The MLE is obtained by minimizing -logL using "nlm", where L is the log-likelihood.

Value

mu_L	mean of L and its standard error
mu_X	mean of X and its standard error
var_L	variance of L and its standard error
var_X	variance of X and its standard error
cov_LX	covariance between L and X and its standard error
c	inclusion probability, defined by c=Pr(L<=X), and its standard error
test	Likelihood ratio statistic and p-value
C	Cramer-von Mises goodness-of-fit test statistics
K	Kolmogorov-Smirnov goodness-of-fit test statistics

Author(s)

Takeshi EMURA

References

Emura T, Konno Y (2012), Multivariate Normal Distribution Approaches for Dependently Truncated Data. Statistical Papers 53 (No.1), 133-149.

Emura T, Konno Y (2014), Erratum to: Multivariate Normal Distribution Approaches for Dependently Truncated Data, Statistical Papers 55 (No.4): 1233-36

Examples

```
l.trunc=c(1,2,3,4,5,6,7,8,8)
x.trunc=c(2,4,4,5,5,7,7,9,10)
PMLE.Normal(l.trunc,x.trunc,testimator=FALSE)
```

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